A Simulator for Hedonic Games

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What's a hedonic game?

A set of players and, for each player, a ranking of possible groups to join

Example of a Hedonic Game

Favorite group

Loathed group

	Austin:	$AG >_A$	$A >_A$	$AGC >_A$	AC
	Dr Goldsmith:	$AG >_G$	$G >_G$	$AGC >_G$	GC
	Cory:	$AKC >_C$	$KC >_C$	$AC >_C$	C

Possible partitions:
{AGC}
{AG, C}
{A, GC}
{AC, G}
{A, G, C}

Basic Notation

$G = (N, \{\succeq_i : i \in N\})$ is a hedonic game.

N is the (finite) set of players.

Each $\succeq_i$ is a ranking of the coalitions containing i.

A class of hedonic games is any (finite or infinite) set of hedonic games.

N = {Austin, Dr G, Cory}

π = {{Austin, Dr G}, {Cory}}

π is a partition of N.

$\pi(i)$ is the coalition in π containing i.

Core stability

A (nonempty) coalition C blocks a partition π iff every player i in C would be happier in C than in $\pi(i)$.

i.e., C blocks π iff $\forall i \in C\colon C >_i \pi(i)$.

π is core stable iff no possible coalition $C \subseteq N$ blocks π.

Core Stability with Austin, Dr G, and Cory

Favorite group

Loathed group

	Austin:	AG $>_A$	A $>_A$	AGC $>_A$	AC
	Dr Goldsmith:	AG $>_G$	G $>_G$	AGC $>_G$	GC
	Cory:	AKC $>_C$	KC $>_C$	AC $>_C$	C

Possible partitions:	Core stable?
{AGC}	No. AG blocks
{AG, C}	Yes!
{A, GC}	No. G blocks
{AC, G}	No. A blocks
{A, G, C}	No. AG blocks

Core Stability with Austin, Dr G, and Cory (version 2)

Favorite group

Loathed group

	Austin:	$AG >_A$	$AC >_A$	$A >_A$	AGC
	Dr Goldsmith:	$GC >_G$	$AG >_G$	$G >_G$	AGC
	Cory:	$AC >_C$	$GC >_C$	$C >_C$	AGC

Possible partitions:	Core stable?
{AGC}	No. AG blocks
{AG, C}	No. GC blocks
{A, GC}	No. AC blocks
{AC, G}	No. AG blocks
{A, G, C}	No. AG blocks

Two frequently-asked questions

Given a class of hedonic games...

1. Is there always a core-stable partition?
2. If not, how hard is it to decide?

Friend-Oriented Hedonic Games

Each player labels every other player as either a friend or an enemy.

Ranking:
More friends is a lot better; **fewer enemies** is a little better.

1. Is there always a core-stable partition? Yes

(Dimitrov, Borm, Hendrickx, Sung. 2006.)

Enemy-Oriented Hedonic Games

Each player labels every other player as either a friend or an enemy.

Ranking:
Fewer enemies is a lot better; **more friends** is a little better.

1. Is there always a core-stable partition? Yes

(Dimitrov, Borm, Hendrickx, Sung. 2006.)

Fractional Hedonic Games

Each player scores every other player. (e.g. Cory ranks Austin 3.46)

Ranking:
Higher **average score** is better.

1. Is there always a core-stable partition? No
2. How hard is it to decide? Σ_2^p -complete!

(Aziz, Brandl, Brandt, Harrenstein, Olsen, Peters. 2017.)

Altruistic Hedonic Games

Each player labels every other player as either a friend or an enemy.

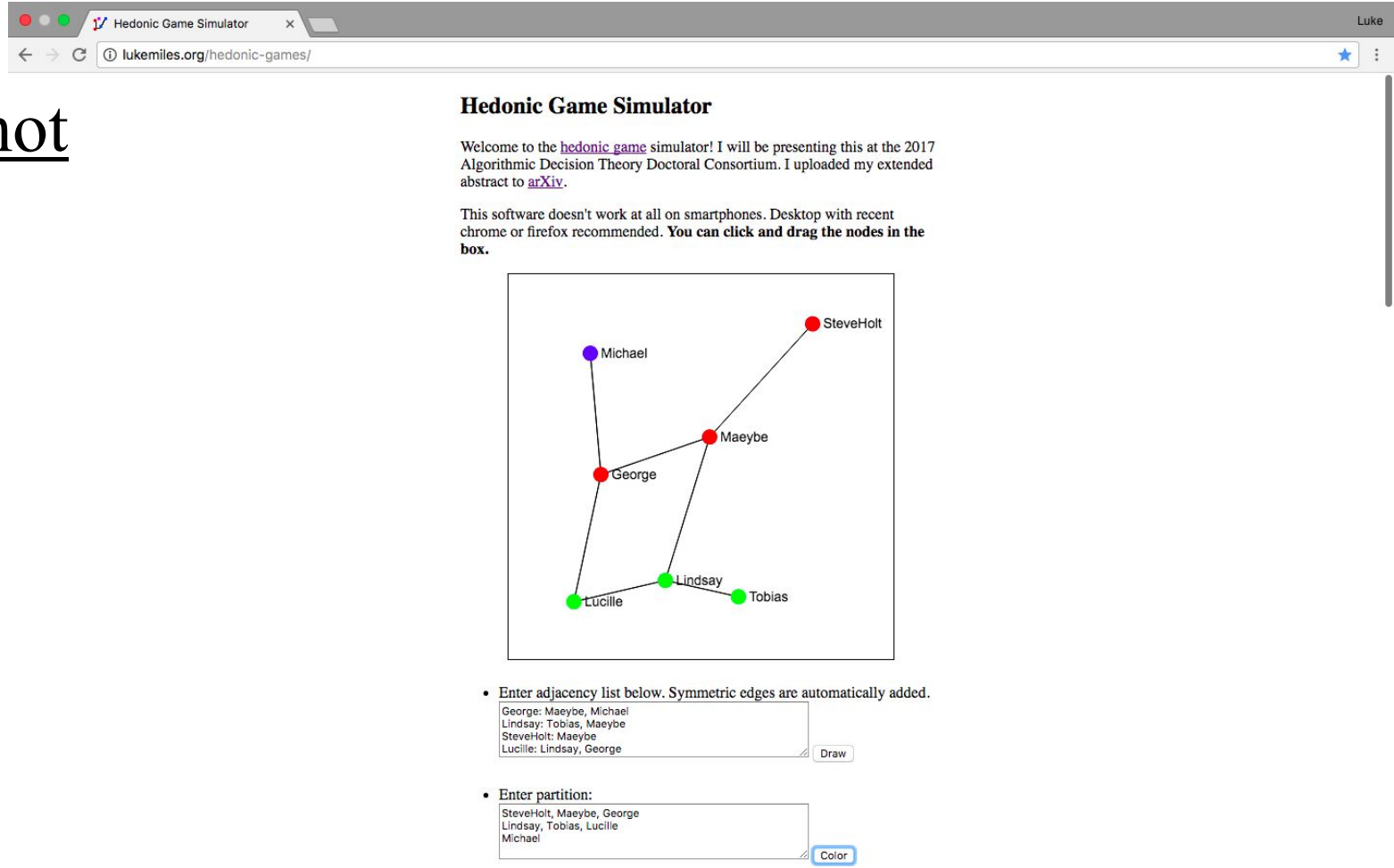
Ranking: “I’ll pick the coalition in which my friends and I are both happy.”

1. Is there always a core-stable partition? No
2. How hard is it to decide? Varies

(Nguyen, Rey, Rey, Rothe, Schend. 2016.)

The Simulator

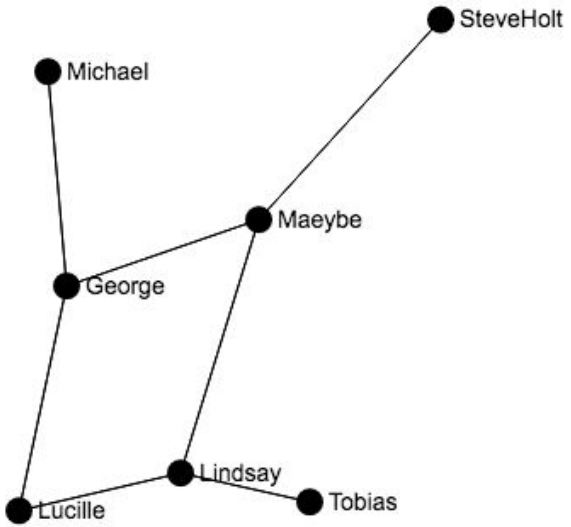
Screenshot


Hedonic Game Simulator
Luke
lukemiles.org/hedonic-games/
Hedonic Game Simulator
Welcome to the hedonic game simulator! I will be presenting this at the 2017 Algorithmic Decision Theory Doctoral Consortium. I uploaded my extended abstract to arXiv.
This software doesn't work at all on smartphones. Desktop with recent chrome or firefox recommended. You can click and drag the nodes in the box.
SteveHolt
Michael
Maeybe
George
Lindsay
Tobias
Lucille
Enter adjacency list below. Symmetric edges are automatically added.
George: Maeybe, Michael
Lindsay: Tobias, Maeybe
SteveHolt: Maeybe
Lucille: Lindsay, George
Draw
Enter partition:
SteveHolt, Maeybe, George
Lindsay, Tobias, Lucille
Michael
Color

Make adjacency list

George: Maeybe, Michael
Lindsay: Tobias, Maeybe
SteveHolt: Maeybe
Lucille: Lindsay, George

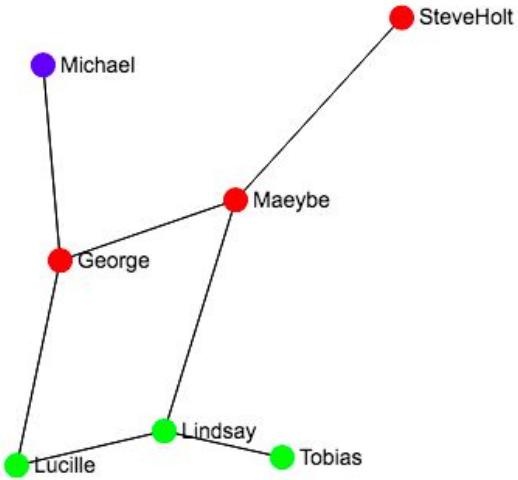
Draw



Make partition

SteveHolt, Maeybe, George
Lindsay, Tobias, Lucille
Michael

Color



Choose Class of Hedonic Games

Lindsay, Tobias, Lucille
Michael

- **Player type:** Set all players to have preferences.

[-]

Choose Stability Notion

$score_i^{FR}(A$

Introduced in [ABH2014].

- **Stability check:** Is this partition ?

Check this partition | Check existence

- individually rational
- Nash-stable
- individually stable
- contractually individually stable
- popular
- strictly popular
- ✓ core-stable
- strictly core-stable
- perfect

No. Counterexample: coalition {George,Michael}

[-]

Compute Scores

- Below you can compute every player's score of every other coalition in the partition. (A player i's score of a coalition C is actually i's score of $C \cup \{i\}$.) Compute [-]

	{SteveHolt,Maeybe,George}	{Lindsay,Tobias,Lucille}	{Michael}	{}
George	0.33	0.25	0.50	0
Lindsay	0.25	0.67	0	0
Lucille	0.25	0.33	0	0
Maeybe	0.67	0.25	0	0
Michael	0.25	0	0	0
SteveHolt	0.33	0	0	0
Tobias	0	0.33	0	0